

Ad Hoc Networking With Cost-Effective Infrastructure: Generalized Capacity Scaling

Cheol Jeong, *Member, IEEE* and Won-Yong Shin, *Member, IEEE*

Abstract

Capacity scaling of a large hybrid network with unit node density, consisting of n wireless *ad hoc* nodes, base stations (BSs) equipped with multiple antennas, and one remote central processor (RCP), is analyzed when wired backhaul links between the BSs and the RCP are *rate-limited*. We deal with a general scenario where the number of BSs, the number of antennas at each BS, and the backhaul link rate can scale at arbitrary rates relative to n (i.e., we introduce three scaling parameters). We first derive the minimum backhaul link rate required to achieve the same capacity scaling law as in the infinite-capacity backhaul link case. Assuming an arbitrary rate scaling of each backhaul link, a generalized achievable throughput scaling law is then analyzed in the network based on using one of pure multihop, hierarchical cooperation, and two infrastructure-supported routing protocols, and moreover, three-dimensional information-theoretic operating regimes are explicitly identified according to the three scaling parameters. In particular, we show the case where our network having a power limitation is also fundamentally in the degrees-of-freedom- or *infrastructure-limited* regime, or both. In addition, a generalized cut-set upper bound under the network model is derived by cutting not only the wireless connections but also the wired connections. It is shown that our upper bound matches the achievable throughput scaling even under realistic network conditions such that each backhaul link rate scales slower than the aforementioned minimum-required backhaul link rate.

Index Terms

Achievability, backhaul link, base station (BS), cut-set upper bound, general capacity scaling, hybrid network, infrastructure, remote central processor (RCP).

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C. Jeong is with the DMC R&D Center, Samsung Electronics, Suwon 443-742, Republic of Korea (E-mail: cheol.jeong@ieee.org).

W.-Y. Shin is with the Department of Computer Science and Engineering, Dankook University, Yongin 448-701, Republic of Korea (E-mail: wyshin@dankook.ac.kr).

I. INTRODUCTION

In the era of the internet of things (IoT), referred to as a world of massive devices equipped with sensors connected to the internet, machine-to-machine (M2M) communications play an important role as an emerging technology. As a large number of M2M devices participate in communications, the protocol design efficiently delivering a number of packets becomes more crucial. While numerical results via computer simulations depend heavily on specific operating parameters for a given protocol, a study on the capacity scaling of a large-scale network provides a fundamental limit on the network throughput and an asymptotic trend with respect to the number of nodes. Hence, one can obtain insights on the practical design of a protocol by characterizing the capacity scaling.

A. Previous Work

Gupta and Kumar's pioneering work [1] introduced and characterized the sum throughput scaling law in a large wireless *ad hoc* network. For the network having n nodes randomly distributed in a unit area, it was shown in [1] that the total throughput scales as $\Theta(\sqrt{n/\log n})$.¹ This throughput scaling is achieved by the nearest-neighbor multihop (MH) routing strategy. In [3]–[5], MH schemes were further developed and analyzed in the network, while their average throughput per source–destination (S–D) pair scales far slower than $\Theta(1)$ —the total throughput scaling was improved to $\Theta(\sqrt{n})$ by using percolation theory [3]; the effect of multipath fading channels on the throughput scaling was studied in [4]; and the trade-off between power and delay was examined in terms of scaling laws in [5]. Together with the studies on MH, it was shown that a hierarchical cooperation (HC) strategy [6], [7] achieves an almost linear throughput scaling, i.e., $\Theta(n^{1-\epsilon})$ for an arbitrarily small $\epsilon > 0$, in the dense network of unit area. As alternative approaches to achieving a linear scaling, novel techniques such as networks with node mobility [8], interference alignment [9], [10], directional antennas [11], and infrastructure support [12] have also been proposed. Capacity scaling was studied when the node density over the area is inhomogeneous [13]–[15]. In [16], [17], a cognitive network consisting of primary and secondary networks was considered for studying the scaling law.

Especially, since long delay and high cost of channel estimation are needed in *ad hoc* networks with only wireless connectivity, the interest in study of more amenable networks using infrastructure support has greatly been growing. Such hybrid networks consisting of both wireless *ad hoc* nodes and infrastructure nodes, or equivalently base stations (BSs), have been introduced and analyzed in [12], [18]–[21]. Using very high frequency bands can be thought of as one of promising ways to meet the high throughput requirements for the next generation communications since abundant frequency resources are available in these bands. A very small wavelength due to the very high frequency bands enables us to deploy a vast number of antennas at each BS (i.e., large-scale multi-antenna systems [22]). In a hybrid network where each BS is equipped with a large number of antennas, the optimal capacity scaling was characterized by introducing two new routing protocols, termed infrastructure-supported single-hop (ISH) and infrastructure-supported multihop (IMH) protocols [21]. In the ISH protocol, all wireless source nodes in each cell communicate with its belonging BS using either a single-hop multiple-access or a single-hop broadcast. In the IMH protocol,

¹We use the following notation: i) $f(x) = O(g(x))$ means that there exist constants C and c such that $f(x) \leq Cg(x)$ for all $x > c$, ii) $f(x) = o(g(x))$ means that $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$, iii) $f(x) = \Omega(g(x))$ if $g(x) = O(f(x))$, iv) $f(x) = w(g(x))$ if $g(x) = o(f(x))$, and v) $f(x) = \Theta(g(x))$ if $f(x) = O(g(x))$ and $g(x) = O(f(x))$ [2].

source nodes in each cell communicate with its belonging BS via the nearest-neighbor MH routing.

In hybrid networks with ideal infrastructure [12], [18]–[21], BSs have been assumed to be interconnected by infinite-capacity wired links. In large-scale ad hoc networks, it is not cost-effective to assume a long-distance optical fiber for all BS-to-BS backhaul links. In practice, it is rather meaningful to consider a cost-effective finite-rate backhaul link between BSs. One natural question is what are the fundamental capabilities of hybrid networks with *rate-limited* backhaul links in supporting n nodes that wish to communicate concurrently with each other. To in part answer this question, the throughput scaling was studied in [23], [24] for a simplified hybrid network, where BSs are connected only to their neighboring BSs via a finite-rate backhaul link—lower and upper bounds on the throughput were derived in one- and two-dimensional networks. However, in [23], [24], the system model under consideration is comparatively simplified, and the form of achievable schemes is limited only to MH routings. In [25], a general hybrid network deploying multi-antenna BSs was studied in fundamentally analyzing how much rate per BS-to-BS link is required to guarantee the optimal capacity scaling achieved for the infinite-capacity backhaul link scenario [21].

More practically, packets arrived at a certain BS in a radio access network (RAN) are delivered to a core network (CN), and then are transmitted from the CN to other BSs in the RAN, while neighboring BSs have an interface through which only signaling information is exchanged between them [26]. The cellular network operating based on a remote central processor (RCP) to which all BSs are connected is well suited to this realistic scenario [27]–[31]. In [31], the set of BSs connected to the RCP via limited-capacity backhaul links was adopted in studying the performance of the multi-cell processing in cooperative cellular systems using Wyner-type models, where an achievable rate for the uplink channel of such a cellular model was analyzed. To the best of our knowledge, characterizing an information-theoretic capacity scaling law of large hybrid networks (i.e. more general than the Wyner-type model) with finite-capacity backhaul links in the presence of the RCP has never been conducted before in the literature.

B. Contribution

In this paper, we introduce a more general hybrid network with unit node density (i.e., a hybrid extended network), consisting of n wireless ad hoc nodes, multiple BSs equipped with multiple antennas, and one RCP, in which wired backhaul links between the BSs and the RCP are *rate-limited*. Our network model is well-suited for the cloud-RAN that has recently received a great deal of attention by assuming that some functionalities of the BSs are moved to a central unit [32]. We take into account a general scenario where three scaling parameters of importance including i) the number of BSs, ii) the number of antennas at each BS, and iii) each backhaul link rate can scale at arbitrary rates relative to n . We first derive the minimum rate of a BS-to-RCP link (or equivalently, an RCP-to-BS link) required to achieve the same capacity scaling law as in the hybrid network with infinite-capacity infrastructure. By showing that hybrid networks can work optimally even with a cost-effective finite-rate backhaul link (but not with the infinite-capacity backhaul link), we can provide a vital guideline to design a *wireless* backhaul that is an important component constituting future fifth generation (5G) networks. Assuming an arbitrary rate scaling of each backhaul link, we then analyze a new achievable throughput scaling law. Inspired by the achievability result in [21], for our achievable scheme, we use one of pure MH, HC, and two different infrastructure-supported routing protocols. Moreover, we identify *three-dimensional* information-theoretic operating regimes explicitly according to the aforementioned three scaling parameters. In each operating

regime, the best achievable scheme and its throughput scaling results are shown. Besides the fact that extended networks of unit node density are fundamentally power-limited [33], we are interested in further finding the case for which our network, having a power limitation, is either in the degrees-of-freedom (DoF)-limited regime, where the performance is limited by the number of BSs or the number of antennas per BS, or in the *infrastructure-limited* regime, where the performance is limited by the rate of backhaul links, or in both. Especially, studying the infrastructure-limited regime would be very challenging when we want to efficiently design the wireless backhaul given the number of BSs and the number of antennas per BS. The infrastructure-limited regime in hybrid networks has never been identified before in the literature. We thus characterize these qualitatively different regimes according to the three scaling parameters.

In addition, a generalized upper bound on the capacity scaling is derived for our hybrid network with finite-capacity infrastructure based on the cut-set theorem. In order to obtain a tight upper bound on the aggregate capacity, we consider two different cuts under the network model. The first cut divides the network area into two halves by cutting the wireless connections. An interesting case is the use of a new cut (i.e., the second cut), which divides the network area into another halves by cutting not only the wireless connections but also the wired connections. It is shown that our upper bound matches the achievable throughput scaling for an arbitrary rate scaling of the backhaul link rate. This indicates that using one of the four routing protocols (i.e., pure MH, HC, ISH, and IMH protocols) can achieve the optimal capacity scaling even in the extended hybrid network with *rate-limited* infrastructure, while the fundamental operating regimes are identified accordingly depending on the backhaul link rate. Hence, it turns out to be order-optimal for all three-dimensional operating regimes of our network.

Our main contribution can be summarized as follows.

- The derivation of the minimum backhaul link rate required to achieve the same capacity scaling law as in the infinite-capacity backhaul link case.
- The analysis of a generalized achievable throughput scaling law assuming an arbitrary rate scaling of each backhaul link.
- The explicit identification of three-dimensional operating regimes according to the number of BSs, the number of antennas at each BS, and the backhaul link rate; and the characterization of both DoF- and infrastructure-limited regimes for our hybrid network which is fundamentally power-limited.
- The derivation of a generalized cut-set upper bound for an arbitrary rate scaling of each backhaul link, resulting in the order optimality of the assumed network.

C. Organization

The rest of this paper is organized as follows. The system and channel models are described in Section II. In Section III, the routing protocols with and without infrastructure support are presented and their transmission rates are shown. In Section IV, the minimum required backhaul link rate is derived, and then a generalized achievable throughput scaling is analyzed. A generalized cut-set upper bound on the capacity scaling is derived in Section V. Finally, Section VI summarizes our paper with some concluding remarks.

D. Notations

Throughout this paper, bold upper and lower case letters denote matrices and vectors, respectively. The superscripts T and \dagger denote the transpose and conjugate transpose, respectively, of a matrix (or a vector). The matrix \mathbf{I}_N is an $N \times N$ identity matrix, $[\cdot]_{ki}$ is the

(k, i) th element of a matrix, and $E[\cdot]$ is the expectation. The positive semidefinite matrix \mathbf{A} is denoted by $\mathbf{A} \succeq 0$.

II. SYSTEM AND CHANNEL MODELS

In an extended network of unit node density, n nodes are uniformly and independently distributed on a square of area n , except for the area where BSs are placed.² We randomly pick S–D pairings, so that each node acts as a source and has exactly one corresponding destination node. Assume that the BSs are neither sources nor destinations. As illustrated in Fig. 1, the network is divided into m square cells of equal area, where a BS with l antennas is located at the center of each cell. The total number of BS antennas in the network is assumed to scale at most linearly with n , i.e., $ml = O(n)$. Note that this assumption about the antenna scaling is feasible due to the massive multiple-input multiple-output (MIMO) (or large-scale MIMO) technology where each BS is equipped with a very large number of antennas, which has recently received a lot of attention. For analytical convenience, the parameters n , m , and l are related according to

$$n = m^{1/\beta} = l^{1/\gamma},$$

where $\beta, \gamma \in [0, 1)$ with a constraint $\beta + \gamma \leq 1$. This constraint is reasonable since the total number of antennas of all BSs in the network does not need to be larger than the number of nodes in the network.

As depicted in Fig. 1, it is assumed that all the BSs are fully interconnected by wired links through one RCP. For simplicity, it is assumed that the RCP is located at the center of the network. The packets transmitted from BSs are received at the RCP and are then conveyed to the corresponding BSs. In the previous works [12], [18]–[21], the rate of backhaul links has been assumed to be unlimited so that the links are not a bottleneck when packets are transmitted from one cell to another. In practice, however, it is natural for each backhaul link to have a finite capacity that may limit the transmission rate of infrastructure-supported routing protocols. In this paper, we assume that each BS is connected to one RCP through an errorless wired link with *finite rate* $R_{\text{BS}} = n^\eta$ for $\eta \in (-\infty, \infty)$. It is also assumed that the BS-to-RCP or RCP-to-BS link is not affected by interference.

The uplink channel vector between node i and BS b is denoted by

$$\mathbf{h}_{bi}^{(u)} = \left[\frac{e^{j\theta_{bi,1}^{(u)}}}{r_{bi,1}^{(u)\alpha/2}}, \frac{e^{j\theta_{bi,2}^{(u)}}}{r_{bi,2}^{(u)\alpha/2}}, \dots, \frac{e^{j\theta_{bi,l}^{(u)}}}{r_{bi,l}^{(u)\alpha/2}} \right]^T, \quad (1)$$

where $\theta_{bi,t}^{(u)}$ represents the random phases uniformly distributed over $[0, 2\pi)$ based on a far-field assumption, which is valid if the wavelength is sufficiently small [3], [21]. Here, $r_{bi,t}^{(u)}$ denotes the distance between node i and the t th antenna of BS b , and $\alpha > 2$ denotes the path-loss exponent. The downlink channel vector between BS b and node i is similarly denoted by

$$\mathbf{h}_{ib}^{(d)} = \left[\frac{e^{j\theta_{ib,1}^{(d)}}}{r_{ib,1}^{(d)\alpha/2}}, \frac{e^{j\theta_{ib,2}^{(d)}}}{r_{ib,2}^{(d)\alpha/2}}, \dots, \frac{e^{j\theta_{ib,l}^{(d)}}}{r_{ib,l}^{(d)\alpha/2}} \right]. \quad (2)$$

²It was shown that the HC scheme is order-optimal in a dense network with infrastructure [21]. In other words, the use of infrastructure is not required in a dense network. Hence, the dense network model is out of interest in our paper where the rate-limited infrastructure is considered.

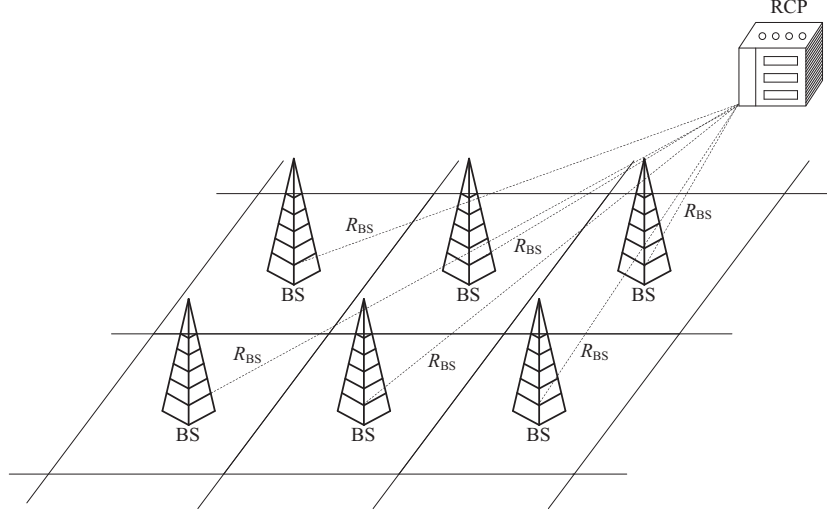


Fig. 1. The hybrid network with limited backhaul link rate R_{BS} between a BS and an RCP.

The channel between nodes i and k is given by

$$h_{ki} = \frac{e^{j\theta_{ki}}}{r_{ki}^{\alpha/2}}. \quad (3)$$

For the uplink-downlink balance, it is assumed that each BS satisfies an average transmit power constraint nP/m , while each node satisfies an average transmit power constraint P . Then, the total transmit power of all BSs is the same as the total transmit power consumed by all wireless nodes. This assumption is based on the same argument as duality connection between multiple access channel (MAC) and broadcast channel (BC) in [34]. The antenna configuration basically follows that of [21]. It is assumed that the antennas of a BS are placed as follows:

- 1) If $l = w(\sqrt{n/m})$ and $l = O(n/m)$, then $\sqrt{n/m}$ antennas are regularly placed on the BS boundary and the remaining antennas are uniformly placed inside the boundary.
- 2) If $l = O(\sqrt{n/m})$, then l antennas are regularly placed on the BS boundary.

Such an antenna deployment guarantees both the nearest neighbor transmission around the BS boundary and the enough spacing between the antennas of each BS. This antenna configuration was adopted in [25], [35]. Let R_n denote the average transmission rate of each source. The total throughput of the network is then defined as $T_n(\alpha, \beta, \gamma, \eta) = nR_n$ and its scaling exponent is given by³

$$e(\alpha, \beta, \gamma, \eta) = \lim_{n \rightarrow \infty} \frac{\log T_n(\alpha, \beta, \gamma, \eta)}{\log n}.$$

III. ROUTING PROTOCOLS WITH AND WITHOUT INFRASTRUCTURE SUPPORT

In this section, routing protocols with and without infrastructure support are illuminated by describing each protocol in detail and showing its achievable transmission rates in each cell.

³To simplify notations, $T_n(\alpha, \beta, \gamma, \eta)$ and $e(\alpha, \beta, \gamma, \eta)$ will be written as T_n and e , respectively, if dropping α , β , γ , and η does not cause any confusion.

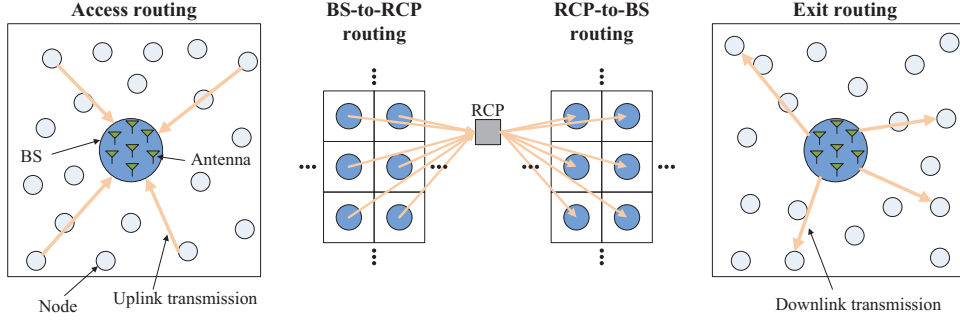


Fig. 2. The ISH protocol. Each square represents a cell in the wireless network.

A. Routing Protocols With Infrastructure Support

The routing protocols supported by BSs having multiple antennas in [21] are described with some modification. In infrastructure-supported routing protocols, the packet of a source is delivered to the corresponding destination of the source using three stages: *access routing*, *backhaul transmission*, and *exit routing*. In the access routing, the packet of a source is transmitted to the home-cell BS. The packet decoded at the home-cell BS is then transmitted to the target-cell BS that is the nearest to the destination of the source via backhaul links (i.e., both BS-to-RCP and RCP-to-BS links). In the exit routing, the target-cell BS transmits the received packet to the destination of the source. Let us start from the following lemma, which quantifies the number of nodes in each cell.

Lemma 1: For $m < n$, the number of nodes in each cell is between

$$\left((1 - \delta_0) \frac{n}{m}, (1 + \delta_0) \frac{n}{m} \right),$$

i.e., $\Theta(n/m)$, with probability larger than $1 - me^{-\Delta(\delta_0)n/m}$, where $\Delta(\delta_0) = (1 + \delta_0) \ln(1 + \delta_0) - \delta_0$ for $0 < \delta_0 < 1$ independent of n .

This lemma can be proved by slightly modifying the proof of [6, Lemma 4.1]. According to the transmission scheme in access and exit routings, the infrastructure-supported routing protocols are categorized into two different protocols as in the following.

1) *ISH Protocol:* There are n/m nodes with high probability (whp) in each cell from Lemma 1. The ISH protocol is illustrated in Fig. 2 and each stage for the ISH protocol is described as follows.

- For the access routing, all source nodes in each cell transmit their packets simultaneously to the home-cell BS via single-hop multiple-access. A transmit power of P is used at each node for uplink transmission.
- The packets of source nodes are then jointly decoded at the BS, assuming that the signals transmitted from the other cells are treated as noise. Each BS performs a minimum mean-square error (MMSE) estimation with successive interference cancellation (SIC). More precisely, the $l \times 1$ unnormalized receive filter \mathbf{v}_i has the expression

$$\mathbf{v}_i = \left(\mathbf{I}_l + \sum_{k>i} P \mathbf{h}_{sk}^{(u)} \mathbf{h}_{sk}^{(u)\dagger} \right)^{-1} \mathbf{h}_{si}^{(u)},$$

which means that the receiver of BS s for the i -th node cancels signals from nodes $1, \dots, i-1$ and treats signals from nodes $i+1, \dots, n/m$ as noise, for every i , when the canceling order is given by $1, \dots, n/m$.

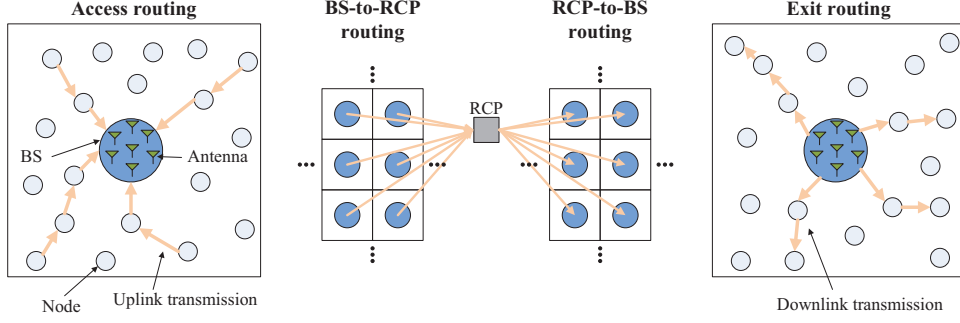


Fig. 3. The IMH protocol. Each square represents a cell in the wireless network.

- In the next stage, the decoded packets are transmitted from the BS to the RCP via BS-to-RCP link.
- The packets received at the RCP are conveyed to the corresponding BS via RCP-to-BS link.
- For the exit routing, each BS in each cell transmits n/m packets received from the RCP, via single-hop broadcast to all the wireless nodes in its cell. The transmitters in the downlink are designed by the dual system of MMSE-SIC receive filters in the uplink, and thus perform an MMSE transmit precoding $\mathbf{u}_1, \dots, \mathbf{u}_{n/m}$ with dirty paper coding at BS s :

$$\mathbf{u}_i = \left(\mathbf{I}_l + \sum_{k>i} p_k \mathbf{h}_{ks}^{(d)\dagger} \mathbf{h}_{ks}^{(d)} \right)^{-1} \mathbf{h}_{is}^{(d)\dagger},$$

where the power $p_k \geq 0$ allocated to each node satisfies $\sum_k p_k \leq \frac{nP}{m}$ for $k = 1, \dots, n/m$. Note that a total transmit power of $\frac{nP}{m}$ is used at each BS for downlink transmission.

2) *IMH Protocol:* Since the extended network is fundamentally power-limited, the ISH protocol may not be effective especially when the node-BS distance is quite long, which motivates us to introduce the IMH protocol. The IMH protocol is illustrated in Fig. 3 and each stage for the IMH protocol is described as follows.

- Each cell is further divided into smaller square cells of area $2 \log n$, termed routing cells. Since $\min\{l, \sqrt{n/m}\}$ antennas are regularly placed on the BS boundary, $\min\{l, \sqrt{n/m}\}$ MH paths can be created simultaneously in each cell.
- For the access routing, the antennas placed only on the BS boundary can receive the packets transmitted from one of the nodes in the nearest-neighbor routing cell. Let us now consider how to set an MH routing path from each source to the corresponding BS. Draw a line connecting a source to one of the antennas of its BS and perform MH routing horizontally or vertically by using the adjacent routing cells passing through the line until its packets reach the corresponding receiver (antenna).
- The BS-to-RCP and RCP-to-BS transmission is the same as the ISH protocol case.
- For the exit routing, each antenna on the BS boundary transmits the packets to one of the nodes in the nearest-neighbor routing cell. Each antenna on the BS boundary transmits its packets via MH transmissions along a line connecting the antenna of its BS to the corresponding destination.

B. Routing Protocols Without Infrastructure Support

The protocols based only on infrastructure support may not be sufficient to achieve the optimal capacity scaling especially when m and l are small. Using one of the MH transmission [1] and the HC strategy [6] may be beneficial in terms of improving the achievable throughput scaling.

1) *MH Protocol*: The MH protocol is described as follows.

- The network is divided into square routing cells of area $2 \log n$.
- Draw a line connecting a source to its destination and perform MH routing horizontally or vertically by using the adjacent routing cells passing through the line until its packets reach the corresponding destination.
- A transmit power of P is used.
- Each routing cell operates the k -time division multiple access scheme to avoid huge interference, where $k > 0$ is some small constant independent of n .

2) *HC Protocol*: The procedure of the HC protocol is as follows.

- The network is divided into clusters each having M nodes.
- (Phase 1) Each source in a cluster transmits its packets to the other $M - 1$ nodes in the same cluster.
- (Phase 2) A long-range MIMO transmission is performed between two clusters having a source and its destination.
- (Phase 3) Each node quantizes the received observations and delivers the quantized data to the rest of nodes in the same cluster. By collecting all quantized observations, each destination can decode its packets.

When each node transmits data within its cluster in Phases 1 and 3, it is possible to apply another smaller-scaled cooperation within each cluster by dividing each cluster into smaller ones. By recursively applying this procedure, it is possible to establish the HC strategy in the network.

C. The Transmission Rates of Routing Protocols

As addressed earlier, the RCP is incorporated into the hybrid network model using BSs. In this subsection, we show how much transmission rates are obtained via wireless links between ad hoc nodes and home-cell BSs for each infrastructure-supported routing protocol. We remark that the transmission rates in both access and exit routings are irrelevant to the rate of backhaul links and thus are essentially the same as the infinite-capacity backhaul link case [21]. The transmission rates of each routing protocol are given in the following lemmas.

Lemma 2 ([21]): Suppose that the ISH protocol is used in the hybrid network of unit node density. Then, the transmission rate in each cell for both access and exit routings is given by

$$\Omega \left(l \left(\frac{m}{n} \right)^{\alpha/2-1} \right). \quad (4)$$

Lemma 3 ([21]): Suppose that the IMH protocol is used in the hybrid network of unit node density. Then, the transmission rate in each cell for both access and exit routings is given by

$$\Omega \left(\min \left\{ l, \left(\frac{n}{m} \right)^{1/2-\epsilon} \right\} \right), \quad (5)$$

where $\epsilon > 0$ is an arbitrarily small constant.

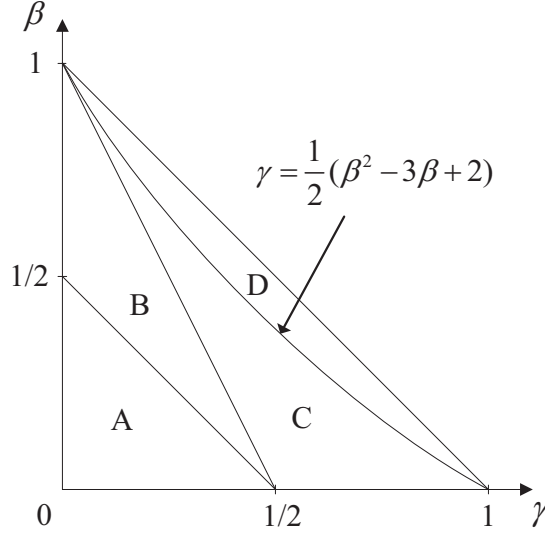


Fig. 4. The operating regimes on the achievable throughput scaling with respect to β and γ for $\eta \rightarrow \infty$.

The total throughput scaling laws achieved by the MH and HC protocols that utilize no infrastructure were derived in [1] and [6], respectively, and are given as follows:

$$T_{n,\text{MH}} = \Omega(n^{1/2-\epsilon})$$

and

$$T_{n,\text{HC}} = \Omega(n^{2-\alpha/2-\epsilon}),$$

where $\epsilon > 0$ is an arbitrarily small constant.

IV. ACHIEVABILITY RESULT

In this section, we first introduce information-theoretic two-dimensional operating regimes with respect to the number of BSs and the number of antennas per BS (i.e., the scaling parameters β and γ) for the infinite-capacity backhaul link scenario. We then derive the minimum rate of each backhaul link, required to achieve the optimal capacity scaling, according to the two-dimensional operating regimes. Assuming that the rate of each backhaul link scales at an arbitrary rate relative to n , we characterize a new achievable throughput scaling, which generalizes the existing achievability result in [21]. The infrastructure-limited regime in which the throughput scaling is limited by the rate of finite-capacity backhaul links is also identified. Furthermore, we closely scrutinize our achievability result according to the *three-dimensional operating regimes* identified by introducing a new scaling parameter η .

A. Two-Dimensional Operating Regimes With Infinite-Capacity Infrastructure

The optimal capacity scaling was derived in [21] for hybrid networks with no RCP when the rate of each BS-to-BS link is unlimited. Although our hybrid network characterized in the presence of RCP differs from the network model in [21], the existing analytical result, including the optimal capacity scaling and information-theoretic operating regimes, can be straightforwardly applied to our network setup for the infinite-capacity backhaul link case, i.e., $\eta \rightarrow \infty$.

TABLE I
ACHIEVABILITY RESULT FOR A HYBRID EXTENDED NETWORK WITH INFINITE-CAPACITY INFRASTRUCTURE [21]

Regime	Condition	Best scheme	Throughput scaling exponent e
A	$2 < \alpha < 3$	HC	$2 - \frac{\alpha}{2}$
	$\alpha \geq 3$	MH	$\frac{1}{2}$
B	$2 < \alpha < 4 - 2\beta - 2\gamma$	HC	$2 - \frac{\alpha}{2}$
	$\alpha \geq 4 - 2\beta - 2\gamma$	IMH	$\beta + \gamma$
C	$2 < \alpha < 3 - \beta$	HC	$2 - \frac{\alpha}{2}$
	$\alpha \geq 3 - \beta$	IMH	$\frac{1+\beta}{2}$
D	$2 < \alpha < \frac{2(1-\gamma)}{\beta}$	HC	$2 - \frac{\alpha}{2}$
	$\frac{2(1-\gamma)}{\beta} \leq \alpha < 1 + \frac{2\gamma}{1-\beta}$	ISH	$1 + \gamma - \frac{\alpha(1-\beta)}{2}$
	$\alpha \geq 1 + \frac{2\gamma}{1-\beta}$	IMH	$\frac{1+\beta}{2}$

As illustrated in Fig. 4, when $\eta \rightarrow \infty$, two-dimensional operating regimes with respect to β and γ are divided into four sub-regimes. To be specific, the best strategy among the four schemes ISH, IMH, MH, and HC depends on the path-loss exponent α and the two scaling parameters β and γ under the network, and the regimes at which the best achievable throughput is determined according to the value of α synthetically constitute our operating regimes. The best scheme and its corresponding scaling exponent $e(\alpha, \beta, \gamma, \infty)$ in each regime are summarized in Table I. The four sub-regimes are described in more detail as follows.

- In Regime A, the infrastructure is not helpful to improve the capacity scaling since β and γ are too small.
- In Regime B, the HC and IMH protocols are used to achieve the optimal capacity scaling. As α increases, the IMH protocol outperforms the HC since long-range MIMO transmissions of the HC becomes inefficient at the high path-loss attenuation regime.
- In Regime C, using the HC and IMH protocols guarantees the order optimality as in Regime B, but leads to a different throughput scaling from that in Regime B.
- In Regime D, the HC protocol has the highest throughput when α is small, but as α increases, the best scheme becomes the ISH protocol. Finally, the IMH protocol becomes dominant when α is very large since the ISH protocol has a power limitation at the high path-loss attenuation regime.

In the next subsections, we shall derive the minimum required rate of backhaul links under each of these operating regimes. In addition, by introducing the scaling parameter η , we shall identify new three-dimensional operating regimes while characterizing a generalized achievable throughput scaling for the hybrid network with finite-capacity infrastructure.

B. The Minimum Required Rate of Backhaul Links

The supportable transmission rate of the backhaul link between BSs and the RCP may increase proportionally with the cost that one needs to pay. In order to give a cost-effective backhaul solution for a large-scale network, we would like to derive the minimum rate scaling of each backhaul link required to achieve the same capacity scaling law as in the infinite-capacity backhaul link case. According to the two-dimensional operating regimes in Fig. 4, the required rate of each BS-to-RCP link (or each RCP-to-BS link), denoted by C_{BS} , is derived in the following theorem.

Theorem 1: The minimum rate of each backhaul link required to achieve the optimal capacity scaling of hybrid networks with infinite-capacity infrastructure is given by

$$C_{BS} = \begin{cases} 0 & \text{for Regime A} \\ \Omega(l) & \text{for Regime B} \\ \Omega\left(\left(\frac{n}{m}\right)^{1/2-\epsilon}\right) & \text{for Regime C} \\ \Omega\left(l\left(\frac{m}{n}\right)^{\log_m(n/l)-1}\right) & \text{for Regime D} \end{cases} \quad (6)$$

for an arbitrarily small constant $\epsilon > 0$. The associated operating regimes with respect to β and γ are illustrated in Fig. 4.

Proof: The required rate of each backhaul link is determined by the multiplication of the number of S–D pairs that transmit packets simultaneously through each link and the transmission rate of the infrastructure-supported routing protocols for each S–D pair. From Table I, no infrastructure-supported protocol is needed in Regime A to achieve the optimal capacity scaling, thereby resulting in $C_{BS} = 0$ in the regime. Let us first focus on the IMH protocol, which is used in Regimes B, C, and D when α is greater than or equal to $4 - 2\beta - 2\gamma$, $3 - \beta$, and $1 + \frac{2\gamma}{1-\beta}$, respectively (see Table I). Let $T_{n,IMH}$ denote the aggregate throughput achieved by the IMH protocol when the rate of each backhaul link is unlimited. Then, from Lemma 3, it follows that $T_{n,IMH} = \Omega\left(m \min\left\{l, \left(\frac{n}{m}\right)^{1/2-\epsilon}\right\}\right)$. Since only $\min\{l, \sqrt{n/m}\}$ nodes among n/m nodes in each cell transmit packets using the IMH protocol, the transmission rate of each activated S–D pair is given by

$$\frac{T_{n,IMH}}{n} \frac{n}{m} \frac{1}{\min\{l, \sqrt{n/m}\}} = \frac{T_{n,IMH}}{m} \frac{1}{\min\{l, \sqrt{n/m}\}}.$$

Note that under the IMH protocol, the number of S–D pairs that transmit packets simultaneously through each backhaul link is $\min\{l, \sqrt{n/m}\}$. Hence, in Regimes B, C, and D, the minimum required rate of each backhaul link to guarantee the throughput $T_{n,IMH}$, denoted by $C_{BS,IMH}$, is given by

$$\begin{aligned} C_{BS,IMH} &= \frac{T_{n,IMH}}{m} \frac{1}{\min\{l, \sqrt{n/m}\}} \min\left\{l, \sqrt{\frac{n}{m}}\right\} \\ &= \Omega\left(\min\left\{l, \left(\frac{n}{m}\right)^{1/2-\epsilon}\right\}\right) \\ &= \begin{cases} \Omega(l) & \text{for Regime B} \\ \Omega\left(\left(\frac{n}{m}\right)^{1/2-\epsilon}\right) & \text{for Regimes C and D,} \end{cases} \end{aligned} \quad (7)$$

which is the same as C_{BS} in Regimes B and C since only the IMH protocol is used between the two infrastructure-supported protocols in these regimes. Now let us turn to the ISH protocol, which is used in Regime D when $\frac{2(1-\gamma)}{\beta} \leq \alpha < 1 + \frac{2\gamma}{1-\beta}$ (see Table I). Let $T_{n,ISH}$ denote the aggregate throughput achieved by the ISH protocol when the rate of each backhaul link is unlimited. Then, from Lemma 2, it follows that $T_{n,ISH} = \Omega\left(ml\left(\frac{m}{n}\right)^{\alpha/2-1}\right)$. Since each S–D pair transmits at a rate $T_{n,ISH}/n$ and the number of S–D pairs that transmit packets simultaneously through each link is n/m , the minimum required rate of each backhaul link for a given α to guarantee the throughput $T_{n,ISH}$, denoted by $C_{BS,ISH}$, is given by

$$C_{BS,ISH} = \frac{T_{n,ISH}}{n} \frac{n}{m} = \frac{T_{n,ISH}}{m} = \Omega\left(l\left(\frac{m}{n}\right)^{\alpha/2-1}\right). \quad (8)$$

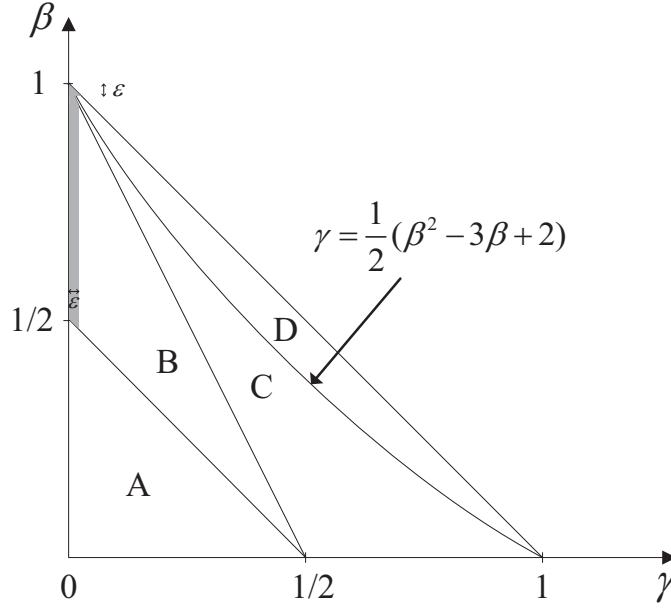


Fig. 5. The operating regime in which $C_{BS} = O(n^\epsilon)$ for an arbitrarily small $\epsilon > 0$ (represented by the shaded area).

Since either the ISH or IMH protocol can be used according to the value of α in Regime D, we should compare the required rates of backhaul links for both protocols. Let us find the minimum required rate C_{BS} with which $\max\{T_{n,IMH}, T_{n,ISH}\}$ can be guaranteed for all α . Using (7) and (8), in Regime D, we have

$$\begin{aligned}
 C_{BS} &= \Omega \left(\max \left\{ \left(\frac{n}{m} \right)^{1/2-\epsilon}, \right. \right. \\
 &\quad \left. \left. \max_{\frac{2(1-\gamma)}{\beta} \leq \alpha < 1 + \frac{2\gamma}{1-\beta}} l \left(\frac{m}{n} \right)^{\alpha/2-1} \right\} \right) \\
 &= \Omega \left(\max \left\{ \left(\frac{n}{m} \right)^{1/2-\epsilon}, l \left(\frac{m}{n} \right)^{(1-\gamma)/\beta-1} \right\} \right) \\
 &= \Omega \left(l \left(\frac{m}{n} \right)^{(1-\gamma)/\beta-1} \right) \\
 &= \Omega \left(l \left(\frac{m}{n} \right)^{\log_m(n/l)-1} \right),
 \end{aligned}$$

where the third equality holds since $\gamma \geq \frac{1}{2}(\beta^2 - 3\beta + 2)$ for Regime D. Therefore, the minimum required rate of each backhaul link, C_{BS} , is finally given by (6), which completes the proof of Theorem 1. \blacksquare

This result indicates that a judicious rate scaling of the BS-to-RCP link (or the RCP-to-BS link) under a given operating regime leads to the order optimality of our general hybrid network along with cost-effective backhaul links.

Remark 1 (Negligibly small backhaul link rates): It is obvious to see that the backhaul is not needed at all in Regime A where using either the MH or HC protocol leads to the best throughput performance of the network. An interesting observation is now to find other regimes in which the minimum required rate of each backhaul link is negligibly small, i.e.,

$C_{\text{BS}} = O(n^\epsilon)$ for an arbitrarily small $\epsilon > 0$. From Theorem 1, it is shown that $C_{\text{BS}} = O(n^\epsilon)$ if $\gamma = \epsilon$ in Regimes B and D or if $\beta = 1 - \epsilon$ in Regimes C and D, which is depicted in Fig. 5. This result reveals that for the case where the number of antennas at each BS is very small or the number of BSs is almost the same as the number of nodes, the backhaul link rate R_{BS} does not need to be infinitely high even for a large number of wireless nodes in the network.

C. Generalized Achievable Throughput Scaling With Finite-Capacity Infrastructure

If the rate of each backhaul link, R_{BS} , is greater than or equal to the minimum required rate C_{BS} in Theorem 1, then the achievable throughput scaling T_n in the hybrid network with finite-capacity infrastructure is the same as the infinite-capacity infrastructure backhaul link scenario. Otherwise, T_n will be decreased accordingly depending on the operating regimes for which the infrastructure-supported routing protocols are used. In this subsection, a generalized achievable throughput scaling is derived with an arbitrary rate scaling of each BS-to-RCP or RCP-to-BS link (or with the scaling parameter $\eta \in (-\infty, \infty)$). The three-dimensional operating regimes with respect to the number of BSs, m , the number of antennas per BS, l , and the backhaul link rate, R_{BS} , are also identified. We start from establishing the following theorem.

Theorem 2: In the hybrid network with the backhaul link rate R_{BS} , the aggregate throughput T_n scales as

$$\Omega \left(\max \left\{ \min \left\{ \max \left\{ ml \left(\frac{m}{n} \right)^{\alpha/2-1}, \right. \right. \right. \right. \\ \left. \left. \left. \min \left\{ ml, m \left(\frac{n}{m} \right)^{1/2-\epsilon} \right\} \right\}, mR_{\text{BS}} \right\}, \right. \\ \left. n^{1/2-\epsilon}, n^{2-\alpha/2-\epsilon} \right\} \right), \quad (9)$$

where $\epsilon > 0$ is an arbitrarily small constant.

Proof: When the rate of each backhaul link is limited by R_{BS} , from (4) and (5), the aggregate rates achieved using the ISH and IMH protocols are given by

$$T_{n,\text{ISH}} = \Omega \left(\min \left\{ ml \left(\frac{m}{n} \right)^{\alpha/2-1}, mR_{\text{BS}} \right\} \right)$$

and

$$T_{n,\text{IMH}} = \Omega \left(\min \left\{ ml, m \left(\frac{n}{m} \right)^{1/2-\epsilon}, mR_{\text{BS}} \right\} \right),$$

respectively, where mR_{BS} represents the maximum supportable rate of backhaul links. We then have

$$\begin{aligned} & \max\{T_{n,\text{ISH}}, T_{n,\text{IMH}}\} \\ &= \Omega \left(\min \left\{ \max \left\{ ml \left(\frac{m}{n} \right)^{\alpha/2-1}, \right. \right. \right. \\ & \quad \left. \left. \left. \min \left\{ ml, m \left(\frac{n}{m} \right)^{1/2-\epsilon} \right\} \right\}, mR_{\text{BS}} \right\} \right) \end{aligned}$$

since $\max\{\min\{a, x\}, \min\{b, x\}\} = \min\{\max\{a, b\}, x\}$. Finally, the achievable total throughput of the network is determined by the maximum of the aggregate rates achieved by the

ISH, IMH, MH, and HC protocols, and thus is given by

$$\begin{aligned}
 T_n &= \max \{T_{n,\text{ISH}}, T_{n,\text{IMH}}, n^{1/2-\epsilon}, n^{2-\alpha/2-\epsilon}\} \\
 &= \Omega \left(\max \left\{ \min \left\{ \max \left\{ ml \left(\frac{m}{n} \right)^{\alpha/2-1}, \right. \right. \right. \right. \\
 &\quad \left. \left. \left. \min \left\{ ml, m \left(\frac{n}{m} \right)^{1/2-\epsilon} \right\} \right\}, mR_{\text{BS}} \right\}, \right. \\
 &\quad \left. n^{1/2-\epsilon}, n^{2-\alpha/2-\epsilon} \right\} \right),
 \end{aligned}$$

which completes the proof of Theorem 2. ■

In the network with rate-limited infrastructure, either the MH and HC protocol may outperform the infrastructure-supported protocols even under certain operating regimes such that using either the ISH or IMH protocol leads to a better throughput scaling for the rate-unlimited infrastructure scenario. This is because the throughput achieved by the ISH and IMH protocols can be severely decreased when the rate R_{BS} becomes the bottleneck. Note that our hybrid extended network is fundamentally power-limited [6]. It is also worth noting that the network may have either a DoF or an infrastructure limitation, or both. In the DoF-limited regime, the performance is limited by the number of BSs or the number of antennas per BS. On the other hand, in the infrastructure-limited regime, the performance is limited by the rate of backhaul links. In the following two remarks, we show the case where our network has such fundamental limitations (the term ϵ is omitted for notational convenience).

Remark 2 (DoF-limited regimes): As seen in Table I, in Regimes B, C, and D, when α is greater than or equal to a certain value, using the ISH or IMH protocol yields the best throughput while the throughput scaling exponent depends on β or γ , or both. In other words, the performance is limited by the number of BSs, m , or the number of antennas per BS, l , or both. Thus, one can say that these high path-loss attenuation regimes are *DoF-limited*.

Remark 3 (Infrastructure-limited regimes): Let us introduce the *infrastructure-limited* regime where the performance is limited by the backhaul link rate R_{BS} ; that is, we show the case where the backhaul links become a bottleneck. In the infrastructure-limited regime, either the ISH or IMH protocol outperforms the other schemes while its throughput scaling exponent depends on η . Two new operating regimes $\tilde{\text{B}}$ and $\tilde{\text{D}}$ causing an infrastructure limitation for some α are identified in Table II. More specifically, Regimes $\tilde{\text{B}}$ and $\tilde{\text{D}}$ become infrastructure-limited when $\alpha \geq 4 - 2\beta - 2\eta$ and $4 - 2\beta - 2\eta \leq \alpha < 2 + \frac{2(\gamma-\eta)}{1-\beta}$, respectively. These regimes are also DoF-limited since the throughput scaling exponent is given by $\beta + \eta$ depending on the number of BSs. In Regime $\tilde{\text{B}}$, the IMH protocol is dominant when $\alpha \geq 4 - 2\beta - 2\eta$.⁴ In Regime $\tilde{\text{D}}$, the following interesting observations are made according to the value of α :

- (High path-loss attenuation regime) If $\alpha \geq 2 + \frac{2(\gamma-\eta)}{1-\beta}$, then the network using the ISH and IMH protocols is limited by the access and exit routings not by the backhaul transmission, and thus achieves the same throughput as that in Regime D.
- (Medium path-loss attenuation regime) If $4 - 2\beta - 2\eta \leq \alpha < 2 + \frac{2(\gamma-\eta)}{1-\beta}$, then the network using the ISH protocol is limited by the backhaul transmission but achieves a higher throughput than those of pure ad hoc routings, which is thus in the infrastructure-limited regime. The network using the IMH protocol is not limited by the backhaul transmission and thus its throughput scaling exponent is always less than $\beta + \eta$.
- (Low path-loss attenuation regime) If $\alpha < 4 - 2\beta - 2\eta$, neither the ISH nor IMH protocol

⁴For some case in Regime $\tilde{\text{B}}$, the network using the ISH protocol is also limited by the backhaul transmission, leading to the same throughput scaling exponent $\beta + \eta$ as the IMH protocol case.

TABLE II
ACHIEVABILITY RESULT FOR A HYBRID EXTENDED NETWORK WITH FINITE-CAPACITY INFRASTRUCTURE

Regime	Range of η	Condition	Best scheme	Throughput scaling exponent e
\tilde{B}	$-\frac{1}{2} \leq \eta < \frac{1}{2}$	$2 < \alpha < 4 - 2\beta - 2\eta$ $\alpha \geq 4 - 2\beta - 2\eta$	HC IMH	$2 - \frac{\alpha}{2}$ $\beta + \eta$
\tilde{D}	$0 \leq \eta < 1$	$2 < \alpha < 4 - 2\beta - 2\eta$ $4 - 2\beta - 2\eta \leq \alpha < 2 + \frac{2(\gamma-\eta)}{1-\beta}$ $2 + \frac{2(\gamma-\eta)}{1-\beta} \leq \alpha < 1 + \frac{2\gamma}{1-\beta}$ $\alpha \geq 1 + \frac{2\gamma}{1-\beta}$	HC ISH ISH IMH	$2 - \frac{\alpha}{2}$ $\beta + \eta$ $1 + \gamma - \frac{\alpha(1-\beta)}{2}$ $\frac{1+\beta}{2}$

can outperform the HC strategy since long-range MIMO transmissions of the HC yields a significant gain for small α .

If η is too small, then some portions in Regimes B, C, and D may turn into Regime A, where the pure ad hoc protocols outperform the infrastructure-supported protocols, which will be specified later. In these regimes, the throughput scaling is not improved even with increasing η and thus the network is not infrastructure-limited. In Regimes B, C, and D, the infrastructure-supported protocols can achieve their maximum throughput, which is the same as the infinite-capacity backhaul link case, since η is sufficiently large. Hence, these three regimes are not fundamentally infrastructure-limited.

The two-dimensional operating regimes specified by β and γ in Fig. 4 can be extended to three-dimensional operating regimes by introducing a new scaling parameter η , where $R_{BS} = n^\eta$ for $\eta \in (-\infty, \infty)$. Since the three-dimensional operating regimes cannot be straightforwardly illustrated and even a three-dimensional representation does not lead to any insight into our analytically intractable network model, we identify the three-dimensional operating regimes by introducing five types of two-dimensional operating regimes, showing different characteristics, with respect to β and γ according to the value of η .

Remark 4 (Three-dimensional operating regimes): The operating regimes with respect to β and γ are plotted in Figs. 6–9 for $\eta < -1/2$, $-1/2 \leq \eta < 0$, $0 \leq \eta < 1/2$, and $1/2 \leq \eta < 1$, respectively. This result is analyzed in Appendix A. In these figures, the infrastructure-limited regimes are marked with a shaded area. Let us closely scrutinize each case.

- $\eta < -\frac{1}{2}$: As shown in Fig. 6, when η is less than $-1/2$, the entire regimes are included in Regime A. This indicates that the infrastructure does not improve the capacity scaling if the backhaul link rate scales slower than $1/\sqrt{n}$, i.e., $R_{BS} = o(1/\sqrt{n})$.
- $-\frac{1}{2} \leq \eta < 0$: As η becomes greater than $-1/2$, the infrastructure can improve the capacity scaling for some cases but the network is limited by the backhaul transmission, thereby resulting in Regime \tilde{B} (see Fig. 7). In Regime \tilde{B} , the HC protocol exhibits the best performance when α is small. As α increases, the throughput achieved by the HC protocol decreases due to the penalty for long-range MIMO transmissions. The IMH protocol via backhaul links then becomes dominant.
- $0 \leq \eta < \frac{1}{2}$: If η is greater than zero, then the infrastructure-supported protocols can fully achieve their throughput as in the network with infinite-capacity infrastructure in Regimes B, C, and D. However, in Regimes \tilde{B} and \tilde{D} , the network using either the ISH or IMH protocol is still limited by the backhaul transmission (specifically when the IMH in Regime \tilde{B} or the ISH in Regime \tilde{D} is used). We refer to Fig. 8.
- $\frac{1}{2} \leq \eta < 1$: As η further increases beyond $1/2$, Regime \tilde{B} disappears since the network using the IMH protocol is not limited by backhaul transmission anymore and the area

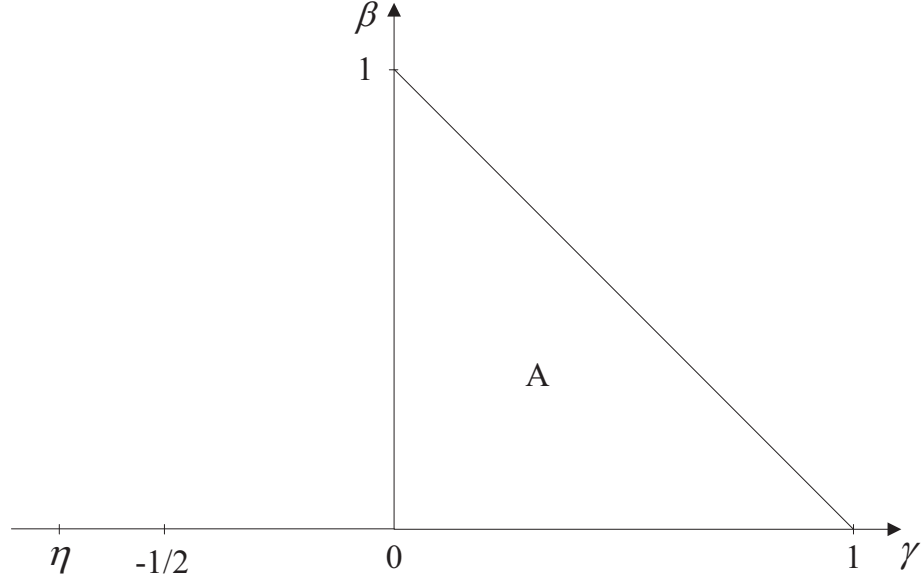


Fig. 6. The operating regime with respect to β and γ , where $\eta < -\frac{1}{2}$.

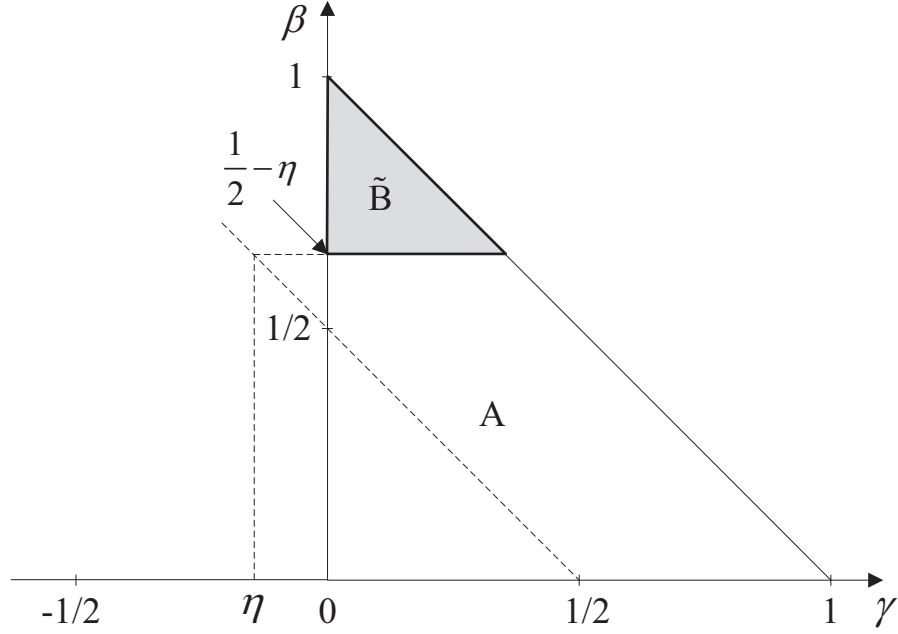


Fig. 7. The operating regimes with respect to β and γ , where $-\frac{1}{2} \leq \eta < 0$.

of Regime \tilde{D} gets reduced (see Fig. 9).

- $\eta \geq 1$: As long as η is greater than or equal to 1, i.e., $R_{BS} = \Omega(n)$, the network has no infrastructure limitation at all, while achieving the same throughput scaling as in the infinite-capacity backhaul link case. The associated operating regimes are then illustrated in Fig. 4.

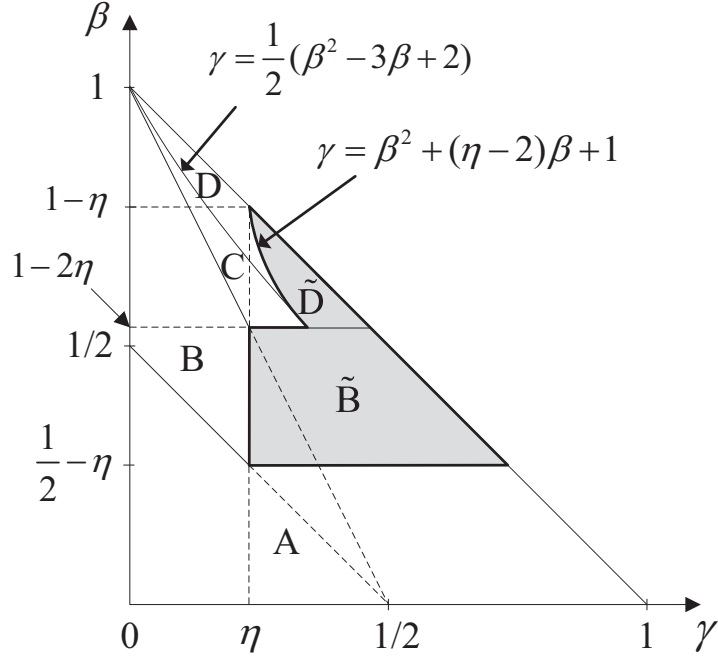


Fig. 8. The operating regimes with respect to β and γ , where $0 \leq \eta < \frac{1}{2}$.

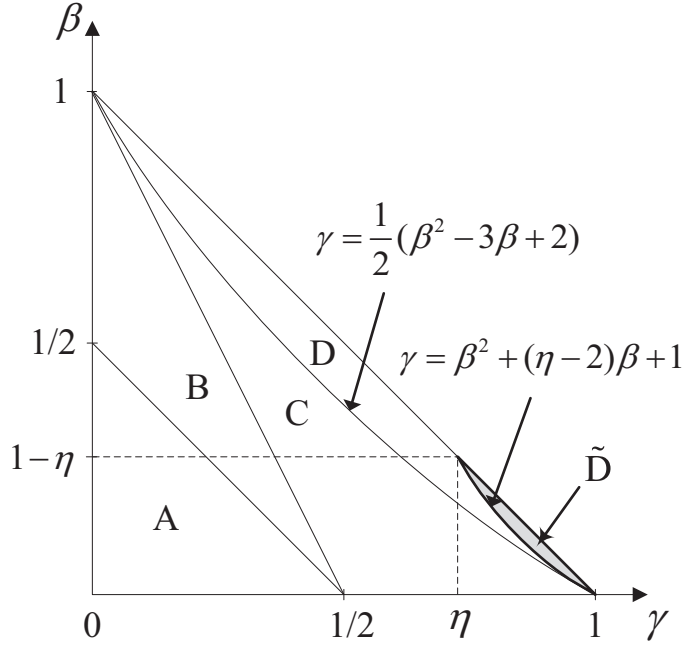


Fig. 9. The operating regimes with respect to β and γ , where $\frac{1}{2} \leq \eta < 1$.

V. CUT-SET UPPER BOUND

In this section, to see how closely our achievable scheme approaches the fundamental limit in the hybrid network with rate-limited BS-to-RCP (or RCP-to-BS) links, a generalized cut-set upper bound on the aggregate capacity scaling based on the information-theoretic approach is derived. As illustrated in Fig. 10, in order to provide a tight upper bound, two cuts L_1

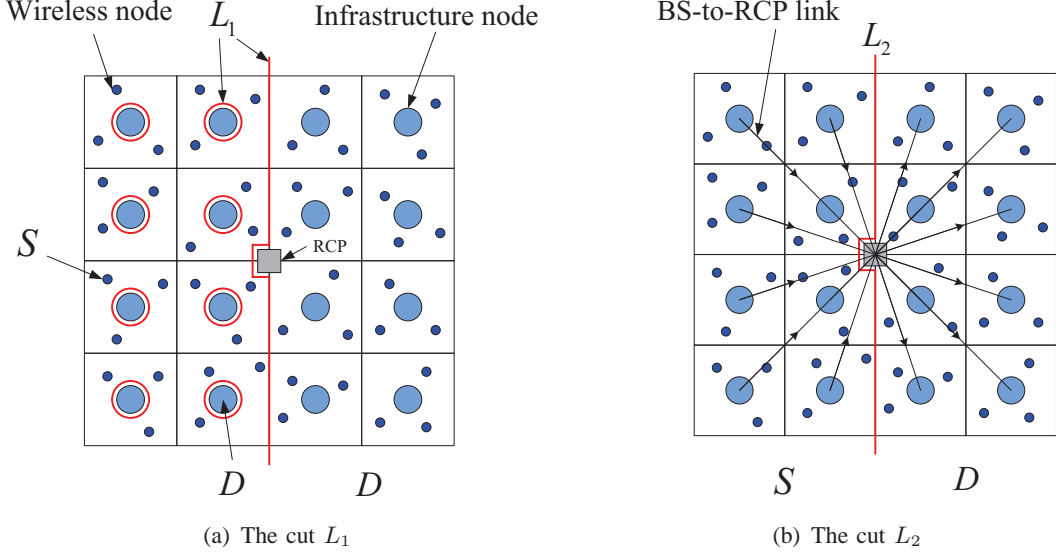


Fig. 10. The cuts L_1 and L_2 in the hybrid extended network. The BS-to-RCP or RCP-to-BS links are not shown in (a) since they are not in effect under L_1 .

and L_2 are taken into account. Similarly as in [21], the cut L_1 divides the network area into two halves by cutting the wireless connections between wireless source nodes on the left of the network and the other nodes, including all BS antennas and one RCP. In addition, to fully utilize the main characteristics of the network with finite-capacity infrastructure, we consider another cut L_2 , which divides the network area into another two halves by cutting the wired connections between BSs and the RCP as well as the wireless connections between all nodes (including BS antennas) located on the left of the network and all nodes (including BS antennas and the RCP) on the right.

Upper bounds obtained under the cuts L_1 and L_2 are denoted by $T_n^{(1)}$ and $T_n^{(2)}$, respectively. By the cut-set theorem, the total capacity is upper-bounded by

$$T_n \leq \min \{T_n^{(1)}, T_n^{(2)}\}. \quad (10)$$

The following two lemmas will be used to derive an upper bound on the capacity in the remainder of this section.

Lemma 4: In our two-dimensional extended network, where n nodes are uniformly distributed and there are m BSs with l regularly spaced antennas, the minimum distance between any two nodes or between a node and an antenna on the BS boundary is greater than $1/n^{1/2+\epsilon_1}$ whp for an arbitrarily small $\epsilon_1 > 0$.

This lemma can be obtained by the derivation similar to that of Lemma 6 in [21].

Lemma 5: Assume a two-dimensional extended network. When the network area with the exclusion of BS area is divided into n squares of unit area, there are less than $\log n$ nodes in each square whp.

The proof of Lemma 5 is given by slightly modifying the proof of in [3, Lemma 1]. Let us first focus on the cut L_1 . Let S_{L_1} and D_{L_1} denote the sets of sources and destinations, respectively, for L_1 in the network. Then, all wireless ad hoc nodes on the left half of the network are S_{L_1} , while all ad hoc nodes on the right half and all BS antennas in the network are destinations D_{L_1} (see Fig. 10(a)). Note that the wired BS-to-RCP (or RCP-to-BS) links do not need to be considered under L_1 since all BSs in the network act as destinations D_{L_1} . In this case, the $\frac{n}{2} \times (\frac{n}{2} + ml)$ MIMO channel between the two sets of nodes and BSs separated

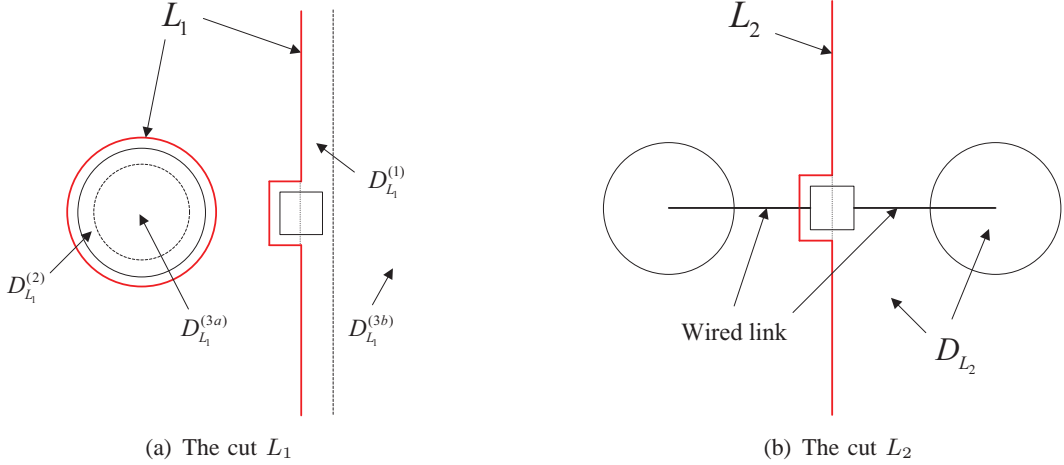


Fig. 11. The partition of destinations under the cuts L_1 and L_2 in the hybrid extended network. To simplify the figure, one and two BSs are shown in (a) and (b) with the RCP, respectively.

by the cut L_1 is formed. The total throughput $T_n^{(1)}$ for sources on the left half is bounded by the capacity of the MIMO channel between S_{L_1} and D_{L_1} , and thus is given by

$$\begin{aligned} T_n^{(1)} &\leq \max_{\mathbf{Q}_{L_1} \geq 0} E \left[\log \det \left(\mathbf{I}_{\frac{n}{2}+ml} + \mathbf{H}_{L_1} \mathbf{Q}_{L_1} \mathbf{H}_{L_1}^\dagger \right) \right] \\ &= \max_{\mathbf{Q}_{L_1} \geq 0} E [\log \det (\mathbf{I}_{\Theta(n)} + \mathbf{H}_{L_1} \mathbf{Q}_{L_1} \mathbf{H}_{L_1}^\dagger)] \end{aligned}$$

where the equality comes from the fact that $n = \Omega(ml)$.⁵ Here, the channel matrix \mathbf{H}_{L_1} consists of the uplink channel vectors $\mathbf{h}_{bi}^{(u)}$ in (1) for $i \in S_{L_1}$, $b \in B$, and h_{ki} in (3) for $i \in S_{L_1}$, $k \in D_r$, where B and D_r denotes the set of BSs in the network and the set of wireless nodes on the right half, respectively. The matrix \mathbf{Q}_{L_1} is the positive semidefinite input covariance matrix whose k th diagonal element satisfies $[\mathbf{Q}_{L_1}]_{kk} \leq P$ for $k \in S_{L_1}$. In order to obtain a tight upper bound, it is necessary to narrow down the class of S-D pairs according to their Euclidean distances. As illustrated in Fig. 11(a), the set D_{L_1} is partitioned into the following four groups according to their locations: $D_{L_1}^{(1)}$, $D_{L_1}^{(2)}$, $D_{L_1}^{(3a)}$, and $D_{L_1}^{(3b)}$. The set $D_{L_1}^{(3a)} \cup D_{L_1}^{(3b)}$ is denoted by $D_{L_1}^{(3)}$. The sets $D_{L_1}^{(1)}$ and $D_{L_1}^{(2)}$ represent the sets of destinations located on the rectangular slab with width one immediately to the right of the centerline (cut) L_1 including the RCP and on the ring with width one immediately inside each BS boundary (cut) on the left half, respectively. The set $D_{L_1}^{(3)}$ is given by $D_{L_1} \setminus (D_{L_1}^{(1)} \cup D_{L_1}^{(2)})$. By generalized Hadamard's inequality [36] as in [6], [37], we then have

$$\begin{aligned} T_n^{(1)} &\leq \max_{\mathbf{Q}_{L_1} \geq 0} E \left[\log \det \left(\mathbf{I}_{\sqrt{n} \log n + 1} + \mathbf{H}_{L_1}^{(1)} \mathbf{Q}_{L_1} \mathbf{H}_{L_1}^{(1)\dagger} \right) \right] \\ &\quad + \max_{\mathbf{Q}_{L_1} \geq 0} E \left[\log \det \left(\mathbf{I}_{O(\sqrt{mn})} + \mathbf{H}_{L_1}^{(2)} \mathbf{Q}_{L_1} \mathbf{H}_{L_1}^{(2)\dagger} \right) \right] \\ &\quad + \max_{\mathbf{Q}_{L_1} \geq 0} E \left[\log \det \left(\mathbf{I}_{\Theta(n)} + \mathbf{H}_{L_1}^{(3)} \mathbf{Q}_{L_1} \mathbf{H}_{L_1}^{(3)\dagger} \right) \right]. \end{aligned} \quad (11)$$

where $\mathbf{H}_{L_1}^{(t)}$ is the matrix with entries $[\mathbf{H}_{L_1}^{(t)}]_{ki}$ for $i \in S_{L_1}$, $k \in D_{L_1}^{(t)}$, and $t = 1, 2, 3$. By analyzing either the sum of the capacities of the multiple-input single-output (MISO) channel

⁵Here and in the sequel, the noise variance is assumed to be one to simplify the notation.

or the amount of power transferred across the network, an upper bound for each term in (11) can be derived. We now establish the following lemma, which shows an upper bound under the cut L_1 .

Lemma 6: Under the cut L_1 in Fig. 11(a), an upper bound on the aggregate capacity, $T_n^{(1)}$, of the hybrid network with rate-limited infrastructure is given by

$$T_n^{(1)} = O \left(n^\epsilon \max \left\{ ml \left(\frac{m}{n} \right)^{\alpha/2-1}, m \min \left\{ l, \sqrt{\frac{n}{m}} \right\}, \sqrt{n}, n^{2-\alpha/2} \right\} \right), \quad (12)$$

where $\epsilon > 0$ is an arbitrarily small constant.

Proof: Let $T_{n,1}^{(1)}$, $T_{n,2}^{(1)}$, and $T_{n,3}^{(1)}$ denote the first to third terms in (11), respectively. By further applying generalized Hadamard's inequality [36], the term $T_{n,1}^{(1)}$ is upper-bounded by

$$\begin{aligned} T_{n,1}^{(1)} &\leq \sum_{k \in D_{L_1}^{(1)}} \log \left(1 + P \sum_{i \in S_{L_1}} |h_{ki}|^2 \right) \\ &\leq c_1 (\sqrt{n} \log n + 1) \log n \leq c_2 n^\epsilon \sqrt{n} \end{aligned} \quad (13)$$

where c_1 and c_2 are some positive constants, independent of n , and $\epsilon > 0$ is an arbitrarily small constant. The second inequality follows from the fact that the minimum distance between any source and destination (including the RCP) is greater than $1/n^{1/2+\epsilon_1}$ whp for an arbitrarily small $\epsilon_1 > 0$ by Lemma 4 and there exist no more than $\sqrt{n} \log n + 1$ nodes in $D_{L_1}^{(1)}$ whp by Lemma 5. Since the remaining terms (i.e., the second and third terms) in (11) can be computed regardless of the presence of the RCP, they are derived by basically following the same approach as that in [21]. From the antenna configuration in our network, the second term in (11), $T_{n,2}^{(1)}$, is bounded by [21]

$$T_{n,2}^{(1)} \leq c_3 m \min \left\{ l, \sqrt{\frac{n}{m}} \right\} \log n \quad (14)$$

where $c_3 > 0$ is some constant independent of n . An upper bound for the third term in (11), $T_{n,3}^{(1)}$, is derived now. If $l = o(\sqrt{n/m})$, there is no destination in $D_{L_1}^{(3a)}$ (see Fig. 11(a)) and thus the information transfer to the set $D_{L_1}^{(3)}$ is the same as that in wireless network with no infrastructure (that is, the information transfer to the set $D_{L_1}^{(3b)}$). Hence, it follows that [6]

$$T_{n,3}^{(1)} = O \left(\max \left\{ n^{2-\alpha/2+\epsilon}, n^{1/2+\epsilon} \right\} \right) \quad (15)$$

If $l = \Omega(\sqrt{n/m})$, using an upper bound for the power transfer from the set S_{L_1} to the set $D_{L_1}^{(3a)}$, the term $T_{n,3}^{(1)}$ is upper-bounded by [21]

$$T_{n,3}^{(1)} \leq \begin{cases} c_4 n^\epsilon \max \left\{ n^{2-\alpha/2}, nl \left(\frac{m}{n} \right)^{\alpha/2} \right\} & \text{if } 2 < \alpha < 3 \\ c_4 n^\epsilon \max \left\{ \sqrt{n}, \frac{n}{\sqrt{l}} \left(\frac{ml}{n} \right)^{\alpha/2} \right\} & \text{if } \alpha \geq 3 \end{cases} \quad (16)$$

where $c_4 > 0$ is some constant independent of n . Using (13)–(16) finally yields (12), which completes the proof of the lemma. \blacksquare

The upper bound $T_n^{(1)}$ matches the achievable throughput scaling within a factor of n^ϵ in the network with infinite-capacity infrastructure ($\eta \rightarrow \infty$), which indicates that $T_n^{(1)}$ does not rely on the parameter η . Note that the first to fourth terms in the max operation of (12)

represent the amount of information transferred to the destination sets $D_{L_1}^{(3a)}$, $D_{L_1}^{(2)}$, $D_{L_1}^{(1)}$, and $D_{L_1}^{(3b)}$, respectively. The third and fourth terms characterize the cut-set upper bound of wireless networks with no infrastructure. By employing infrastructure nodes, it is possible to get an additional information transfer for a given cut L_1 , corresponding to the first and second terms in the max operation of (12).

In addition to the cut L_1 , we now turn to the cut L_2 in Fig. 10(b) to obtain a tight cut-set upper bound in the network with rate-limited infrastructure. Let S_{L_2} and D_{L_2} denote the sets of sources and destinations, respectively, for L_2 in the network. More precisely, under L_2 , all the wireless and infrastructure nodes on the left half are S_{L_2} , while all the nodes on the right half including the RCP are included in D_{L_2} (see Fig. 10(b)). Unlike the case of L_1 , we take into account information flows over the wired connections as well as the wireless connections. In consequence, an upper bound on the aggregate capacity is established based on using the min-cut of our hybrid network, and is presented in the following theorem.

Theorem 3: In the hybrid network with the backhaul link rate R_{BS} , the aggregate throughput T_n is upper-bounded by

$$O \left(\max \left\{ \min \left\{ \max \left\{ n^\epsilon m l \left(\frac{m}{n} \right)^{\alpha/2-1}, \right. \right. \right. \right. \\ \left. \left. \left. n^\epsilon m \min \left\{ l, \sqrt{\frac{n}{m}} \right\} \right\}, m R_{BS} \right\}, \right. \\ \left. n^{1/2+\epsilon}, n^{2-\alpha/2+\epsilon} \right\} \right), \quad (17)$$

where $\epsilon > 0$ is an arbitrarily small constant.⁶

Proof: By the min-cut of the network, the aggregate capacity is upper-bounded by (10). The upper bound on the capacity, $T_n^{(1)}$, under the cut L_1 directly follows from Lemma 6.

An upper bound on the capacity, $T_n^{(2)}$, under the cut L_2 is derived below. Since the cut-set argument under L_1 does not utilize the main characteristics of rate-limited backhaul links, we now deal with a new cut L_2 , which divides the network into two equal halves. A wired link between two BSs that lie on the opposite side of each other from the cut is illustrated in Fig. 11(b). In this case, we get the $\left(\frac{n+ml}{2}\right) \times \left(\frac{n+ml}{2} + 1\right)$ MIMO *wireless* channel and the $\frac{m}{2} \times 1$ MISO *wired* channel between the two sets S_{L_2} and D_{L_2} separated by L_2 . We now derive the amount of information transferred by each channel. Let $T_{n,\text{wireless}}^{(2)}$ and $T_{n,\text{wired}}^{(2)}$ denote the amount of information transferred through the wireless and wired channels under L_2 , respectively. The aggregate capacity obtained under L_2 is then upper-bounded by $T_n^{(2)} \leq T_{n,\text{wireless}}^{(2)} + T_{n,\text{wired}}^{(2)}$.

Let us first focus on deriving $T_{n,\text{wireless}}^{(2)}$, which is bounded by the capacity of the MIMO channel between S_{L_2} and D_{L_2} . It thus follows that

$$\begin{aligned} T_{n,\text{wireless}}^{(2)} &\leq \max_{\mathbf{Q}_{L_2} \geq 0} E \left[\log \det \left(\mathbf{I}_{\frac{n+ml}{2}+1} + \mathbf{H}_{L_2} \mathbf{Q}_{L_2} \mathbf{H}_{L_2}^\dagger \right) \right] \\ &= \max_{\mathbf{Q}_{L_2} \geq 0} E [\log \det (\mathbf{I}_{\Theta(n)} + \mathbf{H}_{L_2} \mathbf{Q}_{L_2} \mathbf{H}_{L_2}^\dagger)], \end{aligned}$$

where the equality comes from $n = \Omega(ml)$. Using the same power transfer argument as [6], an upper bound on $T_{n,\text{wireless}}^{(2)}$ is derived in the following lemma.

Lemma 7: Under the cut L_2 in Fig. 10(b), an upper bound $T_{n,\text{wireless}}^{(2)}$ on the capacity of the

⁶To simplify notations, the terms including ϵ are omitted if dropping them does not cause any confusion.

$\left(\frac{n+ml}{2}\right) \times \left(\frac{n+ml}{2} + 1\right)$ MIMO wireless channel is given by

$$T_{n,\text{wireless}}^{(2)} = O\left(n^\epsilon \max\left\{\sqrt{n}, n^{2-\alpha/2}\right\}\right),$$

where $\epsilon > 0$ is an arbitrarily small constant.

This result can be obtained by straightforwardly applying the proof technique in [6]. This is because the above problem turns into showing the capacity of the classical $n \times n$ MIMO wireless channel formed in pure ad hoc networks due to the fact that $\frac{n+ml}{2} = \Theta(n)$ and the minimum distance between a node and an antenna on the BS boundary is the same as the minimum distance between any two nodes, which thus does not essentially change the result in terms of scaling law.

Next, we turn to analyzing the capacity of the MISO wired channel. There also exist wired links between the two sets separated by L_2 , i.e., BSs on the left half of the network and the RCP placed at the center of the network. From the fact that the $\frac{m}{2}$ BS-to-RCP (or RCP-to-BS) links can be created under L_2 , we have

$$T_{n,\text{wired}}^{(2)} = O(mR_{\text{BS}})$$

since the capacity of each backhaul link is assumed to be R_{BS} . In consequence, under the cut L_2 , it follows that

$$\begin{aligned} T_n^{(2)} &\leq T_{n,\text{wireless}}^{(2)} + T_{n,\text{wired}}^{(2)} \\ &= O\left(\max\left\{mR_{\text{BS}}, n^{1/2+\epsilon}, n^{2-\alpha/2+\epsilon}\right\}\right). \end{aligned} \quad (18)$$

Using (12) and (18), the total throughput T_n is finally upper-bounded by

$$\begin{aligned} T_n &\leq \min\{T_n^{(1)}, T_n^{(2)}\} \\ &= O\left(\min\left\{n^\epsilon \max\left\{ml \left(\frac{m}{n}\right)^{\alpha/2-1}, m \min\left\{l, \sqrt{\frac{n}{m}}\right\}\right\}, \right. \right. \\ &\quad \left. \left. \sqrt{n}, n^{2-\alpha/2}\right\}, \max\{mR_{\text{BS}}, n^{1/2+\epsilon}, n^{2-\alpha/2+\epsilon}\}\right) \\ &= O\left(\max\left\{\min\left\{\max\left\{n^\epsilon ml \left(\frac{m}{n}\right)^{\alpha/2-1}, \right. \right. \right. \right. \\ &\quad \left. \left. \left. n^\epsilon m \min\left\{l, \sqrt{\frac{n}{m}}\right\}\right\}, mR_{\text{BS}}\right\}, n^{1/2+\epsilon}, n^{2-\alpha/2+\epsilon}\right\}\right), \end{aligned}$$

where the last equality follows from $\min\{\max\{a, x\}, \max\{b, x\}\} = \max\{\min\{a, b\}, x\}$. This completes the proof of Theorem 3. \blacksquare

The relationship between the achievable throughput and the cut-set upper bound is now examined as follows.

Remark 5: The upper bound in (17) matches the achievable throughput scaling in Theorem 2 within n^ϵ for an arbitrarily small $\epsilon > 0$ in the hybrid extended network with the finite backhaul link rate R_{BS} . In other words, choosing the best of the four achievable schemes ISH, IMH, MH, and HC is order-optimal for all the operating regimes (even if the rate of each backhaul link between a BS and the RCP is finite). Note that the operating regimes on the upper bound in (17) are basically the same as those on the achievable throughput illustrated in Figs. 6–9. Let us now examine how to achieve each term in (17). The first and second terms in the max operation of (17) correspond to the rate scaling achieved the ISH and IMH protocols for the infinite-capacity backhaul link case (i.e., $\eta \rightarrow \infty$), respectively.

The third term represents the total transmission rate through the backhaul links between all BSs and one RCP when the rate of each link is limited by R_{BS} . The last two terms in the max operation of (17) correspond to the rate scaling achieved by the MH and HC schemes without infrastructure support.

Now we would like to examine in detail the amount of information transfer by each separated destination set.

Remark 6: As mentioned earlier, each term in (17) is associated with one of the destination sets for a given cut. To be concrete, the first and the second terms in the max operation of (17) represent the amount of information transferred to $D_{L_1}^{(3a)}$ and $D_{L_1}^{(2)}$ over the *wireless* connections, respectively. The third term represents the information flows over the *wired* connections from the BSs on the left half to the BSs on the right half. The fourth and fifth terms represent the amount of information transferred to $D_{L_1}^{(1)}$ and $D_{L_1}^{(3b)}$ over the *wireless* connections, respectively.

In Remarks 2 and 3, we showed the case where our network is either in the DoF- or infrastructure-limited regime according to the achievable throughput scaling. In a similar fashion, the identical regimes that lead to such fundamental limitations can also be identified using the cut-set argument drawn under L_1 and L_2 in the following remarks.

Remark 7 (DoF-limited regimes): Similarly as in Remark 2 obtained based on the achievability result, the DoF-limited regime can also be scrutinized by using the cut-set bound result in Theorem 3.

Remark 8 (Infrastructure-limited regimes): In Remark 3 obtained based on the achievability result, the infrastructure-limited regime can also be scrutinized using the upper bound derived under the two cuts.

VI. CONCLUSION

A generalized capacity scaling was characterized for an infrastructure-supported extended network assuming an arbitrary rate scaling of backhaul links. The minimum required rate of each backhaul link was first derived to guarantee the optimal capacity scaling along with a cost-effective backhaul solution. Provided three scaling parameters (i.e., the number of BSs, m , and the number of antennas at each BS, l , the rate of each backhaul link, R_{BS}) scale at arbitrary rates relative to the number of wireless nodes, n , a generalized achievable throughput scaling was then derived based on using one of the two infrastructure-supported routing protocols, ISH and IMH, and the two ad hoc routing protocols, MH and HC. Three-dimensional operating regimes were also explicitly identified according to the three scaling parameters. In particular, we studied the case where our network is fundamentally DoF- or infrastructure-limited, or doubly-limited. Furthermore, to show the optimality of our achievability result, a generalized information-theoretic cut-set upper bound was derived using two cuts. In the hybrid network with rate-limited infrastructure, it was shown that the upper bound matches the achievable throughput scaling for all the three-dimensional operating regimes.

APPENDIX

A. Three-Dimensional Operating Regimes on the Achievable Throughput Scaling

Let e_{ISH} , e_{IMH} , e_{MH} , and e_{HC} denote the scaling exponents for the total throughput achieved by using the ISH, IMH, MH, and HC protocols, respectively. The scaling exponent for the throughput achieved by the ISH and IMH protocols is given by $\beta + \eta$ when the performance is limited by backhaul transmission rate. In the following, the operating regimes with respect to β and γ will be analyzed for each range of the value of η (the terms including ϵ are omitted

for notational convenience). For each case, we will check whether $\max\{e_{\text{ISH}}, e_{\text{IMH}}\}$ is given by $\beta + \eta$. If so, it will be compared with $\max\{e_{\text{MH}}, e_{\text{HC}}\}$ to determine whether the regime is infrastructure-limited or Regime A (i.e., no infrastructure is needed). Otherwise, the regimes will be the same as those for the network with infinite-capacity infrastructure.

1) $\eta < -\frac{1}{2}$ (Fig. 6): In this case, the inequalities $\gamma > \eta$ and $\beta < 1 - 2\eta$ always hold. These inequalities are equivalent to $\beta + \gamma > \beta + \eta$ and $\frac{1+\beta}{2} > \beta + \eta$, respectively. Hence, it follows that $e_{\text{IMH}} = \beta + \eta$ for all the operating regimes, thus resulting in $\max\{e_{\text{ISH}}, e_{\text{IMH}}\} = \max\left\{\min\left\{1 + \gamma - \frac{\alpha(1-\beta)}{2}, \beta + \eta\right\}, \beta + \eta\right\} = \beta + \eta$. Since $\beta + \eta < \max\{e_{\text{MH}}, e_{\text{HC}}\}$ from $\beta + \eta < e_{\text{MH}}$, the entire operating regimes are Regime A as illustrated in Fig. 6.

2) $-\frac{1}{2} \leq \eta < 0$ (Fig. 7): As in Appendix A1 ($\eta < -\frac{1}{2}$), we have $\max\{e_{\text{ISH}}, e_{\text{IMH}}\} = \beta + \eta$ from $\gamma > \eta$ and $\beta < 1 - 2\eta$.

- $\beta < \frac{1}{2} - \eta$: In this case, from $\beta + \eta < e_{\text{MH}}$, it follows that $\beta + \eta < \max\{e_{\text{MH}}, e_{\text{HC}}\}$, and thus the associated operating regimes belong to Regime A.
- $\beta \geq \frac{1}{2} - \eta$: In this case, it follows that $\beta + \eta \geq e_{\text{MH}}$. Hence, the throughput scaling exponent of the best achievable scheme is given by

$$\begin{aligned} \max\{\beta + \eta, e_{\text{MH}}, e_{\text{HC}}\} &= \max\{\beta + \eta, e_{\text{HC}}\} \\ &= \max\left\{\beta + \eta, 2 - \frac{\alpha}{2}\right\}, \end{aligned}$$

and the associated regimes belong to Regime $\tilde{\text{B}}$.

The operating regimes for $-\frac{1}{2} \leq \eta < 0$ are finally illustrated in Fig. 7.

3) $0 \leq \eta < \frac{1}{2}$ (Fig. 8): If $\beta + \gamma < \frac{1}{2}$, then pure ad hoc transmissions without BSs outperform the infrastructure-supported protocols. Thus, the associated operating regimes are included in Regime A. Now, let us focus on the other case, i.e., $\beta + \gamma \geq \frac{1}{2}$, in the following.

- $\gamma > \eta$ and $\beta < 1 - 2\eta$: When $\beta + 2\gamma < 1$, it follows that $e_{\text{IMH}} = \min\{\beta + \gamma, \beta + \eta\} = \beta + \eta$ from $\gamma > \eta$. When $\beta + 2\gamma \geq 1$, it follows that $e_{\text{IMH}} = \min\left\{\frac{1+\beta}{2}, \beta + \eta\right\} = \beta + \eta$ because $\beta < 1 - 2\eta$. Hence, we have $\max\{e_{\text{ISH}}, e_{\text{IMH}}\} = \beta + \eta$. If $\beta < \frac{1}{2} - \eta$ holds, then the associated operating regimes belong to Regime A from the fact that $\beta + \eta < e_{\text{MH}}$. Otherwise, it follows that $\beta + \eta \geq e_{\text{MH}}$, and thus the throughput scaling exponent of the best achievable scheme is given by $\max\left\{\beta + \eta, 2 - \frac{\alpha}{2}\right\}$. In other words, for $\beta \geq \frac{1}{2} - \eta$, the network is infrastructure-limited as α increases beyond a certain value, and the resulting operating regimes belong to Regime $\tilde{\text{B}}$.
- $\gamma < \eta$ and $\beta + 2\gamma < 1$: The throughput scaling exponent achieved by the two infrastructure-supported protocols is given by

$$\begin{aligned} &\max\{e_{\text{ISH}}, e_{\text{IMH}}\} \\ &= \min\left\{\max\left\{1 + \gamma - \frac{\alpha(1-\beta)}{2}, \beta + \gamma\right\}, \beta + \eta\right\} \\ &= \min\{\beta + \gamma, \beta + \eta\} = \beta + \gamma. \end{aligned}$$

Hence, the operating regimes belong to Regime B.

- $\beta + 2\gamma \geq 1$, $\beta \geq 1 - 2\eta$, and $\gamma \geq \beta^2 + (\eta - 2)\beta + 1$: In this case, note that the performance based on the IMH protocol is not limited by backhaul transmission rate since $\beta + \eta \geq \frac{1+\beta}{2}$, or equivalently, $\beta \geq 1 - 2\eta$. Now, let us deal with the ISH protocol. When both conditions $e_{\text{ISH}} = \beta + \eta$ and $\beta + \eta \geq e_{\text{HC}}$ follow, the network is infrastructure-limited for some α , where $\beta + \eta$ is always greater than or equal to e_{MH} (i.e., $\beta \geq \frac{1}{2} - \eta$) in this case. More specifically, we need the condition $4 - 2\beta - 2\eta \leq \alpha < 2 + \frac{2(\gamma - \eta)}{1 - \beta}$

so that the above two conditions hold (see Table II for the details). It thus follows that $\gamma \geq \beta^2 + (\eta - 2)\beta + 1$ from $4 - 2\beta - 2\eta \leq 2 + \frac{2(\gamma - \eta)}{1 - \beta}$. In consequence, the associated operating regimes belong to Regime \tilde{D} .

- $\beta + 2\gamma \geq 1$, $\beta \geq 1 - 2\eta$, and $\gamma < \beta^2 + (\eta - 2)\beta + 1$: In this case, we again note that the performance based on the IMH protocol is not limited by backhaul transmission rate since $\beta \geq 1 - 2\eta$. Now, let us focus on the ISH protocol at the path-loss attenuation regime $\alpha < 2 + \frac{2(\gamma - \eta)}{1 - \beta}$ (see Table II). Then, the throughput scaling exponent $\beta + \eta$ achieved by the ISH protocol is less than $e_{\text{HC}} = 2 - \frac{\alpha}{2}$ since $\alpha < 4 - 2\beta - 2\eta$ due to the fact that $2 + \frac{2(\gamma - \eta)}{1 - \beta} < 4 - 2\beta - 2\eta$. The associated operating regimes are thus the same as those for the network with infinite-capacity infrastructure depicted in Fig. 4.

Finally, the operating regimes for $0 \leq \eta < \frac{1}{2}$ are illustrated in Fig. 8.

4) $\frac{1}{2} \leq \eta < 1$ (Fig. 9): When $\beta + 2\gamma < 1$, it follows that $e_{\text{IMH}} = \beta + \gamma$, which is less than $\beta + \eta$. When $\beta + 2\gamma \geq 1$, we have $e_{\text{IMH}} = \frac{1 + \beta}{2}$, which is less than $\beta + \eta$ since $\beta > 1 - 2\eta$. For this reason, the performance based on the IMH protocol is not limited by backhaul transmission rate for all the operating regimes with respect to β and γ . For the ISH protocol, following the same line as the third bullet of Appendix A3 ($0 \leq \eta < \frac{1}{2}$), we obtain the condition $\gamma \geq \beta^2 + (\eta - 2)\beta + 1$, which is depicted in Regime \tilde{D} . Other regimes are not infrastructure-limited and is the same as those for the network with infinite-capacity infrastructure. The operating regimes for $\frac{1}{2} \leq \eta < 1$ are finally illustrated in Fig. 9.

5) $\eta \geq 1$ (Fig. 4): As in Appendix A4 ($\frac{1}{2} \leq \eta < 1$), the network using the IMH protocol is not limited by the backhaul transmission, i.e., $e_{\text{IMH}} < \beta + \eta$. Furthermore, in this case, there is no operating regime such that both conditions $\gamma \geq \beta^2 + (\eta - 2)\beta + 1$ and $\beta + \gamma \leq 1$ hold. Hence, the entire regimes are not infrastructure-limited, and thus the resulting operating regimes are exactly the same as those for the network with infinite-capacity infrastructure.

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